

R3521

Sub. Code

511301

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2025

Third Semester

Mathematics

CLASSICAL DYNAMICS

(CBCS – 2022 onwards)

Time : 3 Hours

Maximum : 75 Marks

Part A

(10 × 1 = 10)

Answer **all** the questions by choosing the correct option.

- Which of the following best defines a holonomic constraint in a mechanical system? (CO1, K1)
 - A constraint that limits the forces acting on the system
 - A constraint that can be described by an equation involving the coordinates of the system
 - A constraint that limits the velocities of the components
 - A constraint that prevents motion in a particular direction
- Which type of constraint involves forces or contact conditions in a mechanical system? (CO1, K1)
 - Kinematic constraint
 - Holonomic constraint
 - Force constraint
 - Non-holonomic constraint

3. The total energy E of a system can be derived as an integral of motion by summing which two quantities? (CO2, K2)
- (a) Kinetic and potential energies
 - (b) Generalized kinetic energy and generalized potential energy
 - (c) The Lagrangian and the work done by external forces
 - (d) The generalized momenta and generalized velocities
4. When a system has a cyclic coordinate, which of the following remains constant? (CO2, K2)
- (a) The Lagrangian
 - (b) The generalized coordinate q_i
 - (c) The generalized velocity \dot{q}_i
 - (d) The generalized momentum p_i
5. The Hamiltonian principle is a formulation of mechanics based on (CO3, K3)
- (a) Newton's laws of motion
 - (b) The conservation of energy
 - (c) The principle of least action
 - (d) The work-energy theorem
6. In Hamiltonian mechanics, the phase space is described by the space of (CO3, K3)
- (a) Position and time
 - (b) Momentum and time
 - (c) Position and momentum
 - (d) Velocity and momentum

7. The Hamilton-Jacobi equation is a first-order partial differential equation that describes (CO4, K4)
- (a) The evolution of phase space
 - (b) The transformation between coordinates and momenta
 - (c) The motion of a system in terms of the action function
 - (d) The conservation of energy in the system
8. The separability condition for the Hamiltonian system implies (CO4, K4)
- (a) The Hamiltonian can be written as a sum of independent terms involving only position or momentum
 - (b) The generalized coordinates must be periodic in nature
 - (c) The system must have no external forces
 - (d) The action function does not depend on time
9. A canonical transformation in Hamiltonian mechanics preserves (CO5, K5)
- (a) The total energy of the system
 - (b) The equations of motion
 - (c) The form of the Hamiltonian
 - (d) The phase space volume
10. The Lagrangian L is a function of (CO5, K5)
- (a) Position, velocity, and time
 - (b) Position, momentum, and time
 - (c) Position, velocity, and generalized forces
 - (d) Position, momentum, and generalized forces

Part B

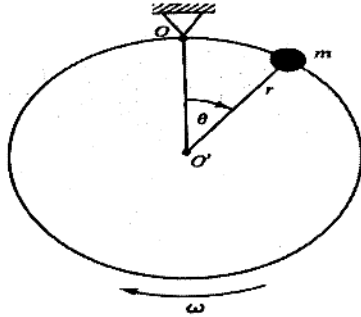
(5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) Explain Nonholonomic Constraints. (CO1, K3)

Or

- (b) Find the kinetic energy and the generalized momentum p_θ for a particle can slide on a rigid wire which is bent in the form of a circle of radius r , the wire rotates about point O on its circumference with an angular velocity ω . (CO1, K4)



12. (a) Show that setting the total time derivative of the energy integral equal to zero gives the same differential equation as multiplying the i th Lagrangian equation by \dot{q}_i and then summing over i . (CO2, K4)

Or

- (b) Explain the Kepler problem in terms of polar coordinates. (CO2, K3)

13. (a) Obtain Hamiltonian's equations of motion for the vertical motion of a particle of mass m in a uniform gravitational field of assuming a damping force of magnitude cv^2 , where v is the velocity and c is the constant. (CO3, K4)

Or

- (b) Explain the derivation of Hamiltonian equation. (CO3, K3)

14. (a) Derive Hamilton-Jacobi equation. (CO4, K3)

Or

- (b) Solve the Hamilton-Jacobi equation with standard Hamiltonian system $H = q + p$ and show that $S = \alpha t + \alpha q - \frac{1}{2}q^2$ is a solution. (CO4, K4)

15. (a) Explain Momentum Transformations. (CO5, K3)

Or

- (b) For a certain canonical transformation $Q = \sqrt{q^2 + p^2}$, $\psi = \frac{1}{2}(q^2 + p^2) \tan^{-1}\left(\frac{q}{p} + \frac{1}{2}qp\right)$ Find $P(q, p)$ and $\phi(q, Q)$. (CO5, K4)

Part C

(5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) A particle of mass m is suspended by a massless wire of length $r = a = b \cos \omega t$ ($a > b > 0$) to form a spherical pendulum, find the equation of motion. (CO1, K5)

Or

- (b) Derive D'Alembert's principle. (CO1, K5)

17. (a) Explain the Routhian function with the illustration. (CO2, K6)

Or

- (b) A particle of mass m can slide on a smooth rigid wire having the form $y = 3x^2$, where a gravity acts in the direction of the negative y -axis
- (i) use the lagrangian method to obtain the equation of motion.
- (ii) Assuming the initial condition $\dot{y}(0) = 0, y(0) = y_0$, find the maximum constraint force during the resulting motion. (CO2, K6)

18. (a) Find the path of minimum length between two given point on the two-dimensional surface of a sphere of radius r . (CO3, K6)

Or

- (b) Explain the Legendre Transformation. (CO3, K5)
19. (a) State and prove Jacobi's theorem. (CO4, K5)

Or

- (b) Use the Hamilton-Jacobi method to solve for x and y as a function of time for a Particle of mass m moves under the action of gravity in the vertical xy plane, where the positive y — axis points vertically upward, assuming that the particle starts at the origin with initial velocity components (\dot{x}_0, \dot{y}_0) . (CO4, K5)

20. (a) Consider the transformation $Q = \log \frac{\sin p}{q}$, $p = q \cot p$.

Obtain the four major types of generating functions associated with this transformation. (CO5, K6)

Or

(b) Find ψ, F_1, F_2, F_3, F_4 for the canonical transformation equations $Q = \sin q$, $P = p - \frac{p - mv_0}{\cos q}$. (CO5, K5)

R3522

Sub. Code

511302

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2025

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2022 onwards)

Time : 3 Hours

Maximum : 75 Marks

Part A

(10 × 1 = 10)

Answer **all** the questions by choosing the correct option.

1. If X is any set, the collection of all one-point subsets of X is a basis for the _____ on X . (CO1, K1)
 - (a) Trivial topology
 - (b) Discrete topology
 - (c) Indiscrete topology
 - (d) Finite Topology
2. A subset A of a topological space X , the _____ of A is defined as the union of all open sets containing in A . (CO1, K1)
 - (a) interior
 - (b) closure
 - (c) limit point
 - (d) continuous
3. If X is a topological space, X is said to be _____ if there exists a metric d on the set X that induces the topology of X . (CO2, K1)
 - (a) metric topology
 - (b) product topology
 - (c) connected
 - (d) metrizable

4. A space is _____ if its only connected subspaces are one-point sets. (CO2, K1)
- (a) Disconnected
 - (b) Compact
 - (c) Totally disconnected
 - (d) Local Compact
5. A space X is said to be _____ if every infinite subset of X has a limit point. (CO3, K1)
- (a) Locally compact
 - (b) sequentially compact
 - (c) countably compact
 - (d) limit point compact
6. Every closed interval in _____ is compact. (CO3, K1)
- (a) \mathbb{Q}
 - (b) \mathbb{C}
 - (c) \mathbb{N}
 - (d) \mathbb{R}
7. A space X is said to be _____ if every subspace of X is normal. (CO4, K1)
- (a) Completely regular
 - (b) Normal
 - (c) Regular
 - (d) Completely normal
8. A subset A of a space X is said to be dense in X if _____.
- (a) $A = X$
 - (b) $\bar{A} = X + 1$
 - (c) $\bar{A} = X$
 - (d) $\bar{X} = A$

9. A _____ of a space X is a compact Hausdorff space Y containing X as a sub-space such that $\overline{X} = Y$. (CO5, K1)
- (a) compactification
 - (b) equivalent
 - (c) manifolds
 - (d) imbedding manifolds
10. A 1-manifold is called a _____. (CO5, K1)
- (a) curve
 - (b) surface
 - (c) area
 - (d) support

Part B

(5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) If A is a subspace of X and B is a subspace of Y , then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.
(CO1, K3)

Or

- (b) Show that every finite point set in a Hausdorff space Y is closed. (CO1, K4)
12. (a) Show that the topologies on \mathbb{R}^n induced by the euclidean metric d and the square metric ρ are the same as the product topology on \mathbb{R}^n . (CO2, K4)

Or

- (b) State and prove Uniform Limit Theorem. (CO2, K3)

13. (a) State and prove extreme value theorem. (CO3, K3)

Or

- (b) Let X be locally compact Hausdorff; let A be a subspace of X . If A is closed in X or open in X , then prove that A is locally compact. (CO3, K3)

14. (a) Show that a subspace of a regular space is regular; a product of regular spaces is regular. (CO4, K4)

Or

- (b) Prove that every compact Hausdorff space is normal. (CO4, K3)

15. (a) If X is a compact m -manifold, then prove that X can be imbedded in \mathbb{R}^n for some positive integer N . (CO5, K3)

Or

- (b) Prove that an arbitrary product of compact spaces is compact in the product topology. (CO5, K4)

Part C

(5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) (i) Let A be a subset of the topological space X ; let A' be the set of all limit points of A . Then prove that $\bar{A} = A \cup A'$. (CO1, K6)
- (ii) State and prove Pasting Lemma.

Or

(b) Let X and Y be topological spaces; let $f: X \rightarrow Y$. Then prove that the following are equivalent:
(CO1, K6)

- (i) f is continuous.
- (ii) For every subset A of X , one has $f(\overline{A}) \subset \overline{f(A)}$
- (iii) For every closed set B of Y , the set $f^{-1}(B)$ is closed in X .
- (iv) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

17. (a) Let $\bar{d}(a, b) = \min\{|a - b|, 1\}$ be the standard bounded metric on \mathbb{R} . If x and y are two points of \mathbb{R}^w , define

$$D(x, y) = \sup \left\{ \frac{\bar{d}(x_i, y_i)}{i} \right\}.$$

Then prove that D is a metric that induces the product topology on \mathbb{R}^w .
(CO2, K6)

Or

(b) State and prove Intermediate value theorem.
(CO2, K5)

18. (a) Prove that the product of finitely many compact spaces is compact.
(CO3, K5)

Or

(b) Let X be a metrizable space. Then prove that the following are equivalent.
(CO3, K6)

- (i) X is compact.
- (ii) X is limit point compact.
- (iii) X is sequentially compact.

19. (a) Prove that every well-ordered set X is normal in the order topology. (CO4, K5)

Or

- (b) State and prove Urysohn metrization theorem. (CO4, K5)

20. (a) State and Prove Tychonoff theorem. (CO5, K5)

Or

- (b) Let $\{U_1, \dots, U_n\}$ be a finite open covering of the normal space X . Then prove that there exists a partition of unity dominated by $\{U_i\}$. (CO5, K6)
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R3523

Sub. Code

511303

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2025

Third Semester

Mathematics

**CALCULUS OF VARIATIONS AND INTEGRAL
EQUATIONS**

(CBCS – 2022 onwards)

Time : 3 Hours

Maximum : 75 Marks

Part A

(10 × 1 = 10)

Answer **all** questions by choosing
the correct option.

1. The characteristic values of λ are inversely proportional to the squares of the _____. (CO1, K2)
(a) Principal axes (b) Semiaxes
(c) Stationary values (d) Characteristic value
2. The derivative of the variation with respect to _____ is same as the variation of derivative. (CO1, K3)
(a) An independent variable
(b) A dependent variable
(c) A variable
(d) A function
3. In one-dimensional calculus, what do we call a maximum and minimum point collectively? (CO2, K2)
(a) Optimum (b) Turning point
(c) Extremum (d) Saddle point

4. The Euler-Lagrange equation is used to find extrema of _____ (CO2, K2)
- (a) Functions (b) Functionals
(c) Vectors (d) Constants
5. $\int_0^{\infty} xe^{-ax} J_1(px) dx = \text{_____}$. (CO3, K2)
- (a) $p(a^2 + p^2)^{-3/2}$ (b) $a(a^2 + p^2)^{-3/2}$
(c) $(a^2 + p^2)^2$ (d) $(a^2 + p^2)^{-3}$
6. The Hankel transform of $\frac{\partial^2 f}{\partial t^2} = \text{_____}$ where f is a function of x and t . (CO3, K3)
- (a) $\frac{\partial^2}{\partial t^2} \tilde{f}(a, t)$ (b) $\frac{\partial^2}{\partial t^2} \tilde{f}(p, t)$
(c) $\frac{\partial^2}{\partial x^2} \tilde{f}(p, t)$ (d) $\frac{\partial^2}{\partial x^2} \tilde{f}(a, t)$
7. In the Fredholm integral equation of the second kind $h(s) = 1, f(s) + \lambda \int_a^b K(s, t) g(t) dt = \text{_____}$. (CO4, K3)
- (a) 0 (b) $h(s)$
(c) $g(s)$ (d) 1
8. A complex-valued function $K(s, t)$ is called _____ if $K(s, t) = K^*(t, s)$. (CO4, K2)
- (a) Faltung (b) Hermitian
(c) Fredholm (d) Linear integral

9. The second iterated kernel $K_2(s,t) = \underline{\hspace{2cm}}$ for the integral equation $g(s) = 1 + \int_0^s stg(t)dt$. (CO5, K1)

- (a) $(s^4t - st^4)$ (b) $(s^4t - st^4)/3$
(c) $(-st^4)/3$ (d) $s^4t/3$

10. The uniqueness of the resolvent kernel $\psi(s,t; \lambda_0) = \underline{\hspace{2cm}}$. (CO5, K2)

- (a) 1 (b) s
(c) 0 (d) t

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Find the minimal surface of revolution passing through the given points of the integral $\frac{1}{2\pi} = \int_{x_1}^{x_2} y(1 + y'^2)^{\frac{1}{2}} dx$. (CO1, K3)

Or

(b) Determine the stationary functions associated with the integral $I = \int_0^1 (Ty'^2 - \rho\omega^2 y^2) dx$ where T, ρ and ω are given constants or functions of x . (CO1, K3)

12. (a) Find the extremals of the functional $\int_0^{\frac{\pi}{2}} ((y')^2 - y^2 + x^2 y) dx$ with $y(0) = -1$ and $y\left(\frac{\pi}{2}\right) = -1$. (CO2, K2)

Or

- (b) Show that the shortest distance between two points in a plane is a straight line. (CO2, K2)

13. (a) State and prove the Parseval's theorem. (CO3, K2)

Or

- (b) Find the Hankel transform of $\frac{1}{r}$ and then apply the inversion formula to get original function. (CO3, K2)

14. (a) Solve the homogenous Fredholm integral equation

$$g(s) = \lambda \int_0^1 e^s e^t g(t) dt. \quad (\text{CO4, K2})$$

Or

- (b) State and prove Fredholm alternative theorem. (CO4, K2)

15. (a) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 e^{s-t} g(t) dt$. (CO5, K3)

Or

- (b) Find the Neumann series for the solution of the integral equations $g(s) = (1+s) + \lambda \int_0^s (s-t)g(t) dt$. (CO5, K2)

Part C

(5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) Determine a continuously differentiable function $y(x)$ for which the integral $I = \int_{x_1}^{x_2} F(x, y, y') dx$ satisfies the end conditions $y(x_1) = y_1, y(x_2) = y_2$.
(CO1, K4)

Or

- (b) Explain about variable endpoints. (CO1, K3)
17. (a) A geodesic on a given surface is a curve, lying on that surface, along which distance between two points is as small as possible. On a plane, a geodesic is a straight line. Determine equations of geodesics on right circular cylinder. (CO2, K4)

Or

- (b) Explain Brachistochrone problem. (CO2, K3)
18. (a) Find the Hankel transform of

$$f(x) = \begin{cases} a^2 - x^2, & 0 < x < a, n = 0 \\ 0, & x > a, n = 0 \end{cases} \quad (\text{CO3, K5})$$

Or

- (b) Prove that $H_v \left[x^v (a^2 - x^2)^{\mu - v - 1} H(a - x); p \right] = 2^{\mu - v - 1} \Gamma(\mu - v) a^\mu p^{v - \mu} J_\mu(pa), a > 0, \mu > v \geq 0$

Hence deduce

$$H_v \left[x^v H(a - x); p \right] = (a^{v+1} / p) J_{v+1}(ap), a > 0 \text{ and}$$

$$H_v \left[\frac{x^v H(a - x)}{\sqrt{(a^2 - x^2)}}; p \right] = \sqrt{\left(\frac{\pi}{2p} \right)} a^{v + \frac{1}{2}} J_{v + \frac{1}{2}}(ap)$$

(CO3, K3)

19. (a) Solve the integral equation (CO4, K4)

$$g(s) = f(s) + \lambda \int_0^1 (1 - 3st)g(t)dt.$$

Or

- (b) Find the approximate solutions of certain integral equation

$$g(s) = e^s - s - \int_0^1 s(e^{st} - 1)g(t)dt. \quad (\text{CO4, K3})$$

20. (a) Solve the integral equation (CO5, K4)

$$g(s) = 1 + \lambda \int_0^\pi [\sin(s+t)]g(t)dt.$$

Or

- (b) Prove the resolvent kernel

$\Gamma\left(s, t; \lambda\right) = \sum_{m=1}^{\infty} \lambda^{m-1} K_m(s, t)$ is absolutely and uniformly convergent for all values of s and t in the circle $|\lambda| < 1/B$. (CO5, K5)

R3524

Sub. Code

511518

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2025

Third Semester

Mathematics

Elective : OPTIMIZATION TECHNIQUES

(CBCS – 2022 onwards)

Time : 3 Hours

Maximum : 75 Marks

Part A

(10 × 1 = 10)

Answer **all** the following objective type questions by choosing the correct option.

1. The Lagrange multiplier method is used to (CO1, K2)
 - (a) Solve differential equations
 - (b) Find saddle points
 - (c) Compute integrals
 - (d) Solve constrained optimization problems
2. The necessary condition for a multivariable function $f(x_1, x_2, \dots, x_n)$ to have an extremum is (CO1, K1)
 - (a) Gradient $\nabla f = 0$
 - (b) Hessian matrix is positive definite
 - (c) Gradient $\nabla f = 1$
 - (d) Function is linear

3. The feasible region in a linear programming problem is (CO2, K2)
- (a) The region satisfying only one constraint
 - (b) The region of intersection of all constraints
 - (c) The region outside all constraints
 - (d) The set of unbounded solutions
4. The simplex method starts at (CO2, K2)
- (a) A feasible solution
 - (b) An optimal solution
 - (c) A solution outside the feasible region
 - (d) A vertex that minimizes cost
5. A stationary point of a function occurs when (CO3, K1)
- (a) Gradient $\nabla f = 0$
 - (b) Hessian matrix is positive definite
 - (c) Objective function is maximized
 - (d) Objective function is minimized
6. Convergence in optimization refers to (CO3, K2)
- (a) The algorithm halting
 - (b) The solution moving closer to the optimum
 - (c) Both (a) and (b)
 - (d) The gradient approaching infinity
7. The feasible direction method ensures (CO4, K1)
- (a) Faster convergence
 - (b) The search moves outside the feasible region
 - (c) The search stays within the feasible region
 - (d) Minimization of the penalty function

8. Slack variables in constrained optimization are used to (CO4, K1)
- (a) Convert inequality constraints to equality constraints
 - (b) Increase the number of variables
 - (c) Simplify the objective function
 - (d) Adjust the feasible region
9. A binary integer programming problem involves decision variables that (CO5, K1)
- (a) Take only positive integer values
 - (b) Take only negative integer values
 - (c) Take values of either 0 or 1
 - (d) Take values between 0 and 1
10. The cutting-plane method is used to (CO5, K2)
- (a) Determine the feasibility of constraints
 - (b) Solve nonlinear programming problems
 - (c) Solve linear programming problems
 - (d) Solve integer programming problems

Part B

(5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Find the second order Taylor's series approximation of the function $f(x_1, x_2, x_3) = x_2^2 x_3 + x_1 e^{x_3}$ about the point $X^* = \{1, 0, -2\}^T$. (CO1, K5)

Or

- (b) Find the extreme points of the function $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$. (CO1, K2)

12. (a) Show that any local minimum solution is global for a linear programming problem. (CO2, K5)

Or

- (b) Solve the following system of equations using pivot operations : $6x_1 - 2x_2 + 3x_3 = 11$, $4x_1 + 7x_2 + x_3 = 21$, $5x_1 + 8x_2 + 9x_3 = 48$. (CO2, K2)

13. (a) Find a suitable scaling (or transformation) of variables to reduce the condition number of the Hessian matrix of the following function to 1 : $f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$. (CO3, K5)

Or

- (b) Write the advantages of Random Search Methods. (CO3, K3)

14. (a) Minimize $f(X) = 9x_1^2 + 6x_2^2 + x_3^2 - 18x_1 - 12x_2 - 6x_3 - 8$ subject to $x_1 + 2x_2 + x_3 \leq 4$, $x_i \geq 0$, $i = 1, 2, 3$. Using the starting point $X_1 = \{0, 0, 0\}^T$, complete one step of sequential linear programming method. (CO4, K4)

Or

- (b) Construct the ϕ_k function, according to interior penalty function methods and plot its contours for the following problem :

Maximize $f = 2x$ subject to $2 \leq x \leq 10$. (CO4, K4)

15. (a) Find the solution for the following using a graphical procedure :

$$\text{Minimize } f = 4x_1 + 5x_2$$

$$\text{subject to } 3x_1 + x_2 \geq 2, \quad x_1 + 4x_2 \geq 5, \quad 3x_1 + 2x_2 \geq 7, \\ x_1, x_2 \geq 0, \text{ integers.} \quad (\text{CO5, K4})$$

Or

- (b) Solve the following mixed integer programming problem using a graphical method :

$$\text{Minimize } f = 4x_1 + 5x_2$$

subject to

$$10x_1 + x_2 \geq 10, \quad 5x_1 + 4x_2 \geq 20,$$

$$3x_1 + 7x_2 \geq 21, \quad x_2 + 12x_2 \geq 12, \quad x_1 \geq 0 \text{ and integer,} \\ x_2 \geq 0. \quad (\text{CO5, K2})$$

Part C

(5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) Find the dimensions of a box of largest volume that can be inscribed in a sphere of unit radius.

(CO1, K4)

Or

- (b) Find the maximum of the function $f(X) = 2x_1 + x_2 + 10$ subject to $g(X) = x_1 + 2x_2^2 = 3$ using the Lagrange multiplier method. Also find the effect of changing the right-hand side of the constraint on the optimum value of f . (CO1, K5)

17. (a) Solve the following by using two phase simplex method :

$$\text{Minimize } f = 2x_1 + 3x_2 + 2x_3 - x_4 + x_5$$

subject to the constraints

$$3x_1 - 3x_2 + 4x_3 + 2x_4 - x_5 = 0,$$

$$x_1 + x_2 + x_3 + 3x_4 + x_5 = 2x_i \geq 0, \quad i = 1 \text{ to } 5. \quad (\text{CO2, K6})$$

Or

- (b) Solve the following Linear Programming problem by the revised simplex method :

$$\text{Minimize } = 15x_1 + 6x_2 + 9x_3 + 2x_4$$

$$\text{subject to } 10x_1 + 5x_2 + 25x_3 + 3x_4 \leq 50,$$

$$12x_1 + 4x_2 + 12x_3 + x_4 \leq 48,$$

$$7x_1 + x_4 \leq 35 \quad x_i \geq 0, \quad i = 1 \text{ to } 4. \quad (\text{CO2, K2})$$

18. (a) Solve the following by Fletcher-Reeves method :

$$\text{Minimize } f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2 \text{ starting}$$

$$\text{from the point } X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (\text{CO3, K3})$$

Or

- (b) Minimize $f = 2x_1^2 + x_2^2$ by using the steepest descent method with the starting point (1, 2). (CO3, K6)

19. (a) Find the dimensions of a rectangular prism-type box that has the largest volume when the sum of its length, width, and height is limited to a maximum value of 60 in and its length is restricted to a maximum value of 36 in. Solve the problem as an unconstrained minimization problem using suitable transformations. (CO4, K4)

Or

- (b) Solve the following problem using an interior penalty function approach coupled with the calculus method of unconstrained minimization :

$$\text{Minimize } f = x^2 - 2x - 1 \quad \text{subject to } 1 - x \geq 0. \quad (\text{CO4, K4})$$

20. (a) Solve the following LP problem using the branch-and-bound method :

$$\text{Maximize } f = 3x_1 + 4x_2$$

$$\text{subject to } 7x_1 + 11x_2 \leq 88, \quad 3x_1 - x_2 \leq 12,$$

$$x_1 \geq 0, x_2 \geq 0, x_i = \text{integer}, i = 1, 2. \quad (\text{CO4, K4})$$

Or

- (b) Solve the following problem using Gomory's cutting plane method :

$$\text{Maximize } f = x_1 + 2x_2$$

$$\text{subject to } x_1 + x_2 \leq 7, \quad 2x_1 \leq 11, \quad 2x_2 \leq 7, \quad x_i \geq 0 \text{ and integer}, i = 1, 2. \quad (\text{CO5, K5})$$